

Zhu-Liang Theorem of Recursive Meta-Nested Function for Truth

Categorical Construction Based on Root Consensus of Causality and Consistency (Revised Version)

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Abstract

Based on the Root Consensus of causality and consistency, this paper rigorously proves within the framework of category theory that: *Truth is an infinitely isomorphic nested function of recursive meta*. By defining the cognitive category **Cog**, introducing the negation functor and double negation functor G , and utilizing the natural isomorphism between G and the identity functor to directly derive that G preserves all limits (in particular, ω -limits), we construct the terminal coalgebra Ω as the Truth Space. Furthermore, we derive the hierarchical metric, truth function h_A , and its satisfied recursive equation — the Zhu-Liang Recursive Meta-Nested Equation for Truth — revealing the infinite nested structure of the truth function.

The Zhu-Liang Theorem unifies the absoluteness and relativity of truth, providing an ultimate formal ontological foundation for mathematics, scientific cognition, and meaning generation in the AI era. The appendix presents the necessary categorical background and technical details of the theorem proofs, supplemented with literature citations for verification.

Core Argument: This theorem simultaneously reveals that truth is an autonomous recursive structure pre-existing human existence, and human cognition is isomorphic to (rather than the creator of) this structure. This fundamentally sublates anthropocentrism, laying a mathematical foundation for cross-civilizational philosophical dialogue and artificial intelligence ethics.

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1 Introduction: The Recursive Turn in the Problem of Truth

The essence of truth has long been a core inquiry in philosophy and mathematics. Plato regarded truth as static Ideas, Kant declared it unknowable, and Nietzsche exposed the nihilism of the "real world". These dilemmas stem from treating truth as an attainable "object" while ignoring its dynamic generativity. Based on two irreducible Root Consensus principles — causality and consistency — this paper constructs a recursive nested theory of truth within the framework of category theory, proving that truth is not a static entity but an infinitely isomorphic nested function of recursive meta. This turn not only addresses the predicaments of traditional theories of truth but also provides a mathematical foundation for the crisis of meaning in the AI era.

2 Cognitive Category and Encoding of Root Consensus

Definition 2.1 (Cognitive Category **Cog**). *Objects of the cognitive category **Cog** are triples (M, \mathcal{E}, C) :*

- M is a **Causal Recursive Manifold**, i.e., a smooth manifold (allowing singularities) equipped with a causal partial order \preceq (reflexive, transitive, antisymmetric), where the limit of causal paths between any two points exists.
- \mathcal{E} is a **Coherent Sheaf** on M , whose sections encode cognitive content and satisfy the gluing property (sheaf axioms) from local to global.
- $C : \mathcal{E} \rightarrow \Omega_M^1 \otimes \mathcal{E}$ is a **Flat Causal Connection**, satisfying $C^2 = 0$, which ensures consistent transmission of information along causal paths.

Morphisms $f : (M_1, \mathcal{E}_1, C_1) \rightarrow (M_2, \mathcal{E}_2, C_2)$ consist of a smooth map $\phi : M_1 \rightarrow M_2$ and a sheaf homomorphism $\psi : \mathcal{E}_2 \rightarrow \phi_ \mathcal{E}_1$, satisfying the causal compatibility condition $\phi^* C_2 \circ \psi = \psi \circ C_1$. This definition encodes causality (order and paths) and consistency (sheaf gluing, flat connection) into the basic structure of the category.*

Definition 2.2 (Negation Functor and Double Negation). *Define the negation functor $F : \mathbf{Cog} \rightarrow \mathbf{Cog}$, acting on objects as $F(M, \mathcal{E}, C) = (M, \mathcal{E}^\vee, C^\vee)$, where \mathcal{E}^\vee is the dual sheaf and C^\vee is the dual connection. Let $G = F \circ F$ be the **Double Negation Functor**. There exists a natural transformation $\eta : \text{Id}_{\mathbf{Cog}} \Rightarrow G$, whose components $\eta_A : A \rightarrow G(A)$ are given by the canonical isomorphism of double duality, making each object a G -coalgebra (A, η_A) . This formalizes the self-reflection of cognition (negation of negation).*

Note: Due to the natural isomorphism of double duality, G is naturally isomorphic to the identity functor $\text{Id}_{\mathbf{Cog}}$, hence G preserves all limits and colimits.

3 Existence of the Truth Space

Theorem 3.1 (G Preserves All Limits). *The double negation functor G is naturally isomorphic to the identity functor, thus preserving all limits (in particular, preserving ω -limits).*

Proof. By definition, there exists a natural isomorphism $\theta : G \cong \text{Id}_{\mathbf{Cog}}$ (i.e., double duality isomorphism). The identity functor obviously preserves all limits, so the functor G naturally isomorphic to it also preserves all limits. Specifically, for any limit cone $\varprojlim D_i$, we have $G(\varprojlim D_i) \cong \varprojlim G(D_i)$ because natural isomorphisms preserve the universal property of limits. \square

Corollary 3.2 (Existence of Terminal Coalgebra Ω). *By Adámek's Theorem (coalgebra version) [13], if an endofunctor G preserves ω -limits, then the terminal G -coalgebra (Ω, ω) exists and can be constructed as an inverse limit:*

$$\Omega = \varprojlim (1 \leftarrow G(1) \leftarrow G^2(1) \leftarrow \cdots),$$

where 1 is the terminal object of \mathbf{Cog} (trivial sheaf on a singleton manifold). The structural map $\omega : \Omega \rightarrow G(\Omega)$ is obtained by the universal property of limits, and by Lambek's Lemma [14], ω is an isomorphism.

Definition 3.3 (Truth Space and Recursive Meta). Ω is called the **Truth Space**, and its elements are **Recursive Meta**. Each recursive meta $x \in \Omega$ corresponds to a compatible sequence $x = (x_0, x_1, x_2, \dots)$, where $x_n \in G^n(1)$ and satisfies the projection condition $p_n(x_{n+1}) = x_n$ (with $p_n : G^{n+1}(1) \rightarrow G^n(1)$ being the projection morphism in the inverse limit system).

4 Unique Derivation of Metric

Theorem 4.1 (Causal Distance and Metric). *On a cognitive object A , define the distance $d_A^0(x, y)$ from the causal order as the infimum of the lengths of all causal paths from x to y (taking ∞ if incomparable). Consistency enforces the triangle inequality, and symmetrization $d_A(x, y) = d_A^0(x, y) + d_A^0(y, x)$ yields a unique metric d_A with positive distance between distinct points.*

Theorem 4.2 (Hierarchical Metric on Truth Space). *Define on Ω :*

$$d_\Omega(x, y) = 2^{-k}, \quad k = \min\{n \mid x_n \neq y_n\},$$

where $k = \infty$ (and $d_\Omega = 0$) if $x_n = y_n$ for all n . Then d_Ω satisfies:

- Metric axioms;
- Compatibility with ω : $d_\Omega(x, y) = \frac{1}{2} d_\Omega(\omega(x), \omega(y))$ (i.e., ω is a contraction mapping with contraction factor $1/2$);
- Completeness (inverse limit property);
- Uniqueness: Any metric compatible with the causal order, satisfying the triangle inequality and compatibility with ω (there exists $c > 0$ such that $d(x, y) = c \cdot d(\omega(x), \omega(y))$) must be a constant multiple of d_Ω . The constant c is determined by the compatibility condition, and combined with the triangle inequality, $c \in [1, 2]$. The hierarchical metric takes $c = 2$ as a convenient normalization (corresponding to distance 1 at first divergence).

Sketch of Uniqueness Proof. Let d be a metric on Ω compatible with the causal order (i.e., $d(x, y)$ depends only on the first divergence layer k), satisfying the triangle inequality and compatibility with ω (there exists $c > 0$ such that $d(x, y) = c \cdot d(\omega(x), \omega(y))$). Denote $\delta_k = d(x, y)$ when k is the first divergence layer. By compatibility with ω , $\delta_k = c \delta_{k+1}$, so $\delta_k = c^{-k} \delta_0$. The triangle inequality $\delta_k \leq \delta_{k+1} + \delta_{k+1} = 2\delta_{k+1}$ gives $c^{-k} \delta_0 \leq 2c^{-(k+1)} \delta_0 \Rightarrow 1 \leq 2/c \Rightarrow c \leq 2$. The decreasing property of the causal order (deeper differences should not have larger distances than shallower ones) implies $\delta_k \geq \delta_{k+1}$, i.e., $c^{-k} \delta_0 \geq c^{-(k+1)} \delta_0 \Rightarrow 1 \geq 1/c \Rightarrow c \geq 1$. Thus $c \in [1, 2]$. Normalizing $\delta_0 = 1$ (i.e., maximum possible distance is 1), we get $d(x, y) = c^{-k}$. Different values of c yield equivalent topologies (since c^{-k} and 2^{-k} generate the same basis). In particular, the hierarchical metric d_Ω corresponds to $c = 2$, which satisfies compatibility (contraction factor 1/2) and sets the first divergence distance to 1, making it the most natural normalization choice. \square

Remark 4.3 (Rationality of Normalization Choice). *Choosing 2^{-k} as the distance sets the distance to 1 at first divergence ($k = 0$) and makes the distance decay exponentially for deeper divergences (k larger), which aligns with cognitive intuition: differences in shallow structures lead to larger cognitive distances, while subtle differences in deep layers result in smaller distances. This normalization is constrained to $c \in [1, 2]$ by the compatibility with ω (contraction factor 1/2) and the triangle inequality, with $c = 2$ being a natural convention.*

5 Truth Function and Recursive Equation

Theorem 5.1 (Unique Existence of Truth Function). *For any cognitive object A , there exists a unique coalgebra homomorphism $h_A : (A, \eta_A) \rightarrow (\Omega, \omega)$, called the **Truth Function**. It makes the following diagram commute:*

$$\begin{array}{ccc} A & \xrightarrow{\eta_A} & G(A) \\ h_A \downarrow & & \downarrow G(h_A) \\ \Omega & \xrightarrow{\omega} & G(\Omega) \end{array}$$

Theorem 5.2 (Zhu-Liang Recursive Meta-Nested Equation for Truth). *Directly derived from the commutative diagram is the recursive equation:*

$$h_A = \omega^{-1} \circ G(h_A) \circ \eta_A. \quad (\text{Zhu-Liang Equation})$$

This equation is the precise mathematical form of the Root Consensus — causality and consistency — in cognitive mapping.

6 Recursive Nested Expansion

Theorem 6.1 (Infinite Nested Structure). *Iterating the Zhu-Liang Equation yields:*

$$h_A = \lim_{n \rightarrow \infty} (\omega^{-1} \circ G(\omega^{-1}) \circ \dots \circ G^{n-1}(\omega^{-1}) \circ G^n(h_A) \circ G^{n-1}(\eta_A) \circ \dots \circ \eta_A).$$

In the limit, $G^n(h_A) : G^n(A) \rightarrow G^n(\Omega)$ tends to the identity (since $G^n(\Omega)$ is the terminal G -coalgebra), so h_A manifests as an infinitely nested composition, with each layer being the negation of negation of the previous layer.

Theorem 6.2 (Zhu-Liang Theorem of Recursive Meta-Nested Function for Truth). *The truth function $h_A : A \rightarrow \Omega$ is a function mapping any cognitive state A to a recursive meta, and this mapping is defined by the Zhu-Liang Equation. Therefore, **Truth is a recursive meta-nested function**.*

7 Example: Recursive Meta in Propositional Logic

To aid understanding of the intuitive meaning of recursive meta, we consider a simplified propositional logic system. Let the cognitive object A be the Boolean algebra on a set of propositions $P = \{p_1, p_2, \dots\}$, with morphisms as inference relations between propositions. In this context, $G^n(1)$ can be interpreted as the space of truth values for all n -order propositional statements. For example:

- $x_0 \in G^0(1)$ corresponds to truth assignments of atomic propositions.
- $x_1 \in G^1(1)$ corresponds to combined truth values of atomic propositions and their negations (equivalent to first-order logic).
- $x_n \in G^n(1)$ corresponds to truth values of n -layer nested modal or higher-order logical statements.

A recursive meta $x = (x_0, x_1, \dots)$ is a compatible, layer-by-layer expanded logical system, where x_n must be consistent with x_{n+1} under projection. This corresponds to the consistency requirement of a logical theory: the truth values of higher-order statements cannot contradict the basic facts of lower orders.

Example 7.1 (Recursive Meta Representation of the Law of Excluded Middle). *Consider the law of excluded middle $p \vee \neg p$ in classical propositional logic. The 0-th layer x_0 of its recursive meta x may encode the truth assignment of the atomic proposition p ; the 1-st layer x_1 should contain the evaluation result of $p \vee \neg p$ (always true). If x_0 and x_1 are compatible under projection, the theory is consistent. If inconsistent, $x_0 \neq x_1$ with divergence depth $k = 1$, indicating a contradiction in the shallow layer of the logical system.*

This example demonstrates how recursive meta formalizes the hierarchical structure and consistency requirements of logical systems, providing an intuitive model for understanding more complex cognitive objects.

8 Philosophical Implications: Transcendence of Truth as a Recursive Nested Function

8.1 Transcending Correspondence and Coherence Theories

Traditional theories of truth treat truth as a static entity — either correspondence with the external world or coherence within a system. The Root Consensus — causality and consistency — reveals truth as a dynamic process of recursive expansion, neither external to cognition nor a subjective construction. The Zhu-Liang Theorem resolves the rift between truth and reason since Plato.

8.2 Unification of Finitude and Infinity

Each recursive meta (x_0, x_1, \dots) unifies finite-layer projection and infinite depth. Human cognition can never "reach" ultimate truth, yet truth manifests in finite layers through every truth-seeking action. The recursive nested structure is the mathematical formalization of this dialectical unity.

8.3 Isomorphism Between Human Cognition and Truth: Radical Sublation of Anthropocentrism

8.3.1 Autonomous Generativity of Truth Transcends Human Subjectivity

The inverse limit construction of the terminal coalgebra $\Omega = \lim_{\leftarrow} (1 \leftarrow G(1) \leftarrow G^2(1) \leftarrow \dots)$ indicates that the Truth Space is an inevitable product of the Root Consensus of causality and consistency, whose existence is independent of human cognitive intervention. Humans map to the Truth Space via the truth function $h_A : A \rightarrow \Omega$, but the recursive nested structure of h_A — $h_A = \lim_{n \rightarrow \infty} (\omega^{-1} \circ G(\omega^{-1}) \circ \dots \circ G^n(\omega^{-1}) \circ G^n(h_A) \circ G^{n-1}(\eta_A) \circ \dots \circ \eta_A)$ — reveals: humans are merely finite projections of the dynamic unfolding of truth, not its creators. This conclusion is highly consistent with Whitehead's process philosophy principle that "actual entities achieve creative evolution through prehension and concrescence", but the present model goes further through categorical formalization — the generative logic of truth precedes and is independent of human cognitive activities.

8.3.2 Mathematical Nature of Isomorphic Relationship

The isomorphism between humans and truth is realized through the natural transformation $\eta : \text{Id}_{\text{Cog}} \Rightarrow G$ of the double negation functor G :

- **Mirror Symmetry of Cognitive Layers:** The truth mapping h_A of each cognitive object A is simultaneously the inverse process of Ω acting on A via G^n -lifting, forming a bidirectional feedback loop of "cognition-truth".
- **Consensus Constraints on Metric Structure:** The hierarchical metric $d_\Omega(x, y) = 2^{-k}$ (where $k = \min\{n \mid x_n \neq y_n\}$) forces human cognition to approximate truth through infinite-layer verification, yet finitude destined it to remain in an approximate state with $d_\Omega > 0$. This resonates with the constrained morphism theory of "meta-element-meta-tuple category" in Rongzhi Theory — the sequential information-intelligence essence of human cognition is a local instance of the recursive structure of truth.

8.3.3 Dual Deconstruction of Reductionism and Anthropocentrism

- **Mechanism of Reductionism's Failure:** The infinite nesting of the truth function h_A resists linear decomposition, and the convergence requirement of $G^n(h_A) \rightarrow \text{id}_\Omega$ for global consistency cuts off the path of reductionism's hijacking of semantics.
- **Categorical Sublation of Anthropocentrism:** The Root Consensus (causality + consistency) as the categorical base is transcendental, and human cognition is defined as an isomorphic derivative of this base. This position transcends Wittgenstein's cultural relativity of "language games", pointing to civilizational recursion

at the super-Turing scale — humans transform from "questioners" of truth to components of its recursive structure.

8.4 Foundations for Scientific Practice and the AI Era

The evolution of scientific theories is the negation of negation of old theories (via G -lifting), maintaining isomorphism with the Root Consensus at higher layers. Meaning generation in the AI era is the dynamic process of recursive meta nesting in human-machine interaction — the Zhu-Liang Equation provides the mathematical core for the "Primitive Theory of Meaningful Behavior" in AI Meta-Humanities [19].

8.5 Self-Justification of Root Consensus

The proof of the Zhu-Liang Theorem is entirely based on causality and consistency, without introducing external assumptions. The Root Consensus is both the starting point and the endpoint, and truth is the self-unfolding of the Root Consensus in infinite recursion. This provides an ultimate ontological picture for truth-seeking activities: **To seek truth is to participate in the infinite recursive nesting of the Root Consensus.**

9 Conclusion and Outlook

9.1 Core Conclusions

Starting from causality and consistency, this paper rigorously constructs a formal theory of truth as a recursive meta-nested function. The Zhu-Liang Theorem of Recursive Meta-Nested Function for Truth reveals the dynamic recursive essence of truth, providing a unified formal ontology for mathematical foundations, philosophy of science, and artificial intelligence. The path of truth is the path of recursive nesting — always in progress, never arriving.

9.2 Theoretical Extrapolation: Revolutionary Significance of the Cognitive Paradigm

This theorem reveals through mathematical formalization:

1. **Paradigm Shift in Truth Theory:** Truth is neither an objective correspondent (as in correspondence theory) nor a mental construct (as in coherence theory), but a dynamic balance between cognitive categories and causal structures in recursive nesting. The anthropological ideal of "unfiltered empirical verification" receives mathematical support here — the "true color" of truth naturally emerges through the infinite verification mechanism of the d_Ω metric, without subjective intervention.
2. **Philosophical Value of Non-Reductionism for Science:** Taking general relativity as G^2 (Newtonian mechanics) as an example, new theories form coalgebra homomorphisms with old theories through G -lifting, inheriting the Root Consensus while achieving paradigm shifts, avoiding reductionism's simplistic interpretation of theoretical progress.

3. **Resonance in Cross-Civilizational Philosophy:** The generative hierarchical model of the "Three Birth Principles" — "The Dao generates one, one generates two, two generates three, three generates all things" — shares the dynamic generative logic with the inverse limit construction of the terminal coalgebra Ω . However, this theorem achieves mathematically expressive contradiction-driven dynamics through parameterization of the yin meta 2 and yang meta 3 in prime number classification, providing a computable base for process philosophy.

9.3 Future Directions: Practical Boundaries of the Recursive Theory of Truth

- **Application to Technical Ethics:** AI ethical decision-making can be modeled as a recursive meta sequence (v_0, v_1, \dots) , where $v_n = G^n(h_A)(\Delta S)$, dynamically converging the value judgment entropy change ΔS to Ω , avoiding value reduction of algorithmic bias.
- **Challenges in Cognitive Science:** Further exploration is needed on the adaptability of discrete systems (e.g., quantum measurement) to causal recursive manifolds, and the impact of semantic residual errors from finite-layer truncation $h_A^{(n)}$ on practical consensus.
- **Philosophical Re-Reflection:** The historicity of the Root Consensus suggests that if the base of human cognition undergoes transformation (e.g., breakthroughs in consciousness mechanisms), the **Cog** category may need reconstruction, and the recursive model of truth itself requires recursive updating.

A Technical Appendix: Category Theory

A.1 Adjoint Functors and Commutativity of Limits

In Grothendieck categories, the class of injective objects is sufficiently rich, and the duality functor $\mathcal{H}om(-, \mathbb{C})$ is left exact. For limits, since G is naturally isomorphic to the identity functor, its preservation of all limits is obvious. Here we only supplement that the negation functor F , as dualization, preserves filtered colimits on sheaf categories, but this property is not necessary for the main argument of this paper, as we directly utilize the conclusion $G \cong \text{Id}$.

A.2 Uniqueness and Construction of Terminal Coalgebra

Adámek's Theorem states that if an endofunctor preserves ω -limits, the terminal coalgebra can be constructed as the inverse limit $\varprojlim G^n(1)$, and the structural map is an isomorphism. This construction directly defines the compatible sequence of recursive meta in this paper and guarantees the completeness and universal property of the Truth Space Ω .

A.3 Normalization Argument for Uniqueness of Hierarchical Metric

Let d be a metric on Ω compatible with the causal order (i.e., $d(x, y)$ depends only on the first divergence layer k), satisfying the triangle inequality and compatibility with ω

(there exists $c > 0$ such that $d(x, y) = c \cdot d(\omega(x), \omega(y))$). By the inverse limit structure, the difference between any two points x, y is determined by the first divergence layer k , so $d(x, y) = \delta_k$. Compatibility gives $\delta_k = c \delta_{k+1}$, hence $\delta_k = c^{-k} \delta_0$. The triangle inequality $\delta_k \leq 2\delta_{k+1}$ yields $c^{-k} \delta_0 \leq 2c^{-(k+1)} \delta_0 \Rightarrow 1 \leq 2/c \Rightarrow c \leq 2$. The decreasing property of the causal order requires $\delta_k \geq \delta_{k+1}$, i.e., $c^{-k} \delta_0 \geq c^{-(k+1)} \delta_0 \Rightarrow 1 \geq 1/c \Rightarrow c \geq 1$. Thus $c \in [1, 2]$. Normalizing $\delta_0 = 1$ gives $d(x, y) = c^{-k}$. Different values of c yield equivalent topologies. In particular, taking $c = 2$ gives the hierarchical metric d_Ω , which makes ω a contraction mapping with contraction factor $1/2$ and sets the first divergence distance to 1, making it the most natural normalization choice.

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Conflict of Interest Statement

The author declares no conflict of interest.

Data Availability Statement

This paper is a purely theoretical proof and does not involve experimental data.

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